MATH 100 Readiness Check

Calculus will likely be one of the most challenging courses you will take in your first year university program. In order to be successful:

- You must have a *strong and recent* algebra and trigonometry background.
- You need to have a clear and realistic idea about the commitment of time and energy necessary.

The intent of this readiness check test is to help you decide whether you are ready for Math 100 or whether you should first refresh your math skills by taking the precalculus course Math 115 (Note that this course does carry University Transfer Credit and can be used for student loan purposes!).

- If you struggle with most of the problems or do not remember a large proportion of this material, you are probably not ready for Math 100 and will want to consider registering in Math 115.
- If you can work through most of the questions but struggle with some of them then you are most likely ready for Math 100. You should be prepared to devote some time to decide whether to stay in Math 100 or transfer to Math 115. Please talk with your instructor to help you choose the right course and strategy for you.

Section A: Order of operations

A.1 Evaluate the following expressions for the given values of the variables without using a calculator:

1. $\frac{-3x^2-2xy^3}{3x^2y^2-2y}$ if $x=-\frac{2}{3}$ $y=\frac{1}{2}$
2. $6x(8-x^2)^{\frac{4}{3}}-8x^2(8-x^2)^{\frac{1}{3}}$ if $x=-4$
3. $\frac{x^{-1}+y^{-1}}{x^{-1}-y^{-2}}$ if $x=4$ $y=2$

A.2 Simplify the following expressions by using an appropriate method. Give your answer using only positive exponents.

1. $5-3(2x+3)^2-(3x+1)(x-4)$
2. $\sqrt{x^2+4}+\sqrt{9x^2+36}$
3. $\frac{1}{x+3+h}-\frac{1}{x+3}$
4. $(\ ^{1}\quad 2^{\ 1})^{\ 2}$

Section C: Analytic Geometry and Basic Graphing

C.1 The Distance, Midpoint, Slope and Point-Slope Formulas

1. For the pair of points $(3,-4)$, $(-2,1)$

D.3 For each pair of functions, evaluate: $f(x)g(x)$, $f(g(x))$ and $g(f(x))$:

1. $f(x) = 2x^2 + 5$, $g(x) = \sqrt{2x-1}$

2. $f(x) = \dfrac{2}{3x-4}$, $g(x) = 8 - 6x$

Find functions $f(x)$ and $g(x)$ such that $k(x) = f(g(x))$ for the following:
(There are different correct answers for each question.)

3. $k(x) = \sqrt{9x^2 + 25}$

4. $k(x) = \dfrac{1}{\sqrt[5]{(x^3 - 27)^2}}$

D.4 Construct a function as needed for each of the following problems:

1. A piece of wire is 50 cm long and is cut into two pieces. The pieces are then used to form a square and a circle. If the length of the piece used to form the circle has a length of *x* cm, then write the total combined area *A* of the circle and the square as a function of *x*.
2. The surface area of a cylinder is given by 2.2. ^{2}

Note that some of the above identities are interrelated, for example:

$$^{2}\theta \qquad ^{2}\theta \qquad \frac{^{2}\theta}{^{2}\theta} \quad \frac{^{2}\theta}{^{2}\theta} \quad \frac{}{^{2}\theta} \qquad ^{2}\theta \quad 1 \quad \sec^{2}\theta$$

ANSWERS

Section C.2

Section D.2

1. $-\frac{10}{3}$
2. $f(a-3)=a^2-8a+20$
3. $3x^2+3xh+h^2$
4. $\dfrac{2}{(4-x-h)(4-x)}$
5. $\dfrac{-2}{\sqrt{1-2x-2h}+\sqrt{1-2x}}$

Section D.3

1. $f(x)g(x)=(2x^2+5)\sqrt{2x-1}$; $f(g(x))=4x+3$, $x\geq\frac{1}{2}$; $g(f(x))=\sqrt{4x^2+9}$
2. $f(x)g(x)=-4$, $f(g(x))=\dfrac{1}{10-9x}$, $g(f(x)=\dfrac{24x-44}{3x-4}$
3. $f(x)=\sqrt{x}$, $g(x)=9x^2+25$
4. $f(x)=x^{-2/5}$, $g(x)=x^3-27$

Section D.4

1. Radius of the circle is $r=\dfrac{x}{2\pi}$ and the length of a side of the square is $s=\dfrac{50-x}{4}$.

   So, $A(x)=\pi\left(\dfrac{x}{2\pi}\right)^2+\left(\dfrac{50-x}{4}\right)^2=\dfrac{x^2}{4\pi}+\dfrac{x^2-100x+2500}{16}$

2. $500=\pi r^2h \Rightarrow h=\dfrac{500}{\pi r^2}$

   Therefore: $S(r)=2\pi r\left(\dfrac{500}{\pi r^2}\right)+2\pi r^2 \Rightarrow S(r)=\dfrac{1000}{r}+2\pi r^2$

3. $A(x)=2x-\frac{2}{3}x^2$

Section E.1

1. $210°$
2. 0.06 radians
3. $-\dfrac{\sqrt{3}}{2}=-0.866$
4. -3.236
5. $45°=\dfrac{\pi}{4}$ rads
6. $53.13°=0.927$ rads
7. $\theta=\dfrac{5\pi}{4}+2\pi k$, $\theta=\dfrac{7\pi}{4}+2\pi k$
8. $\theta=2.214+2\pi k$, $\theta=4.069+2\pi k$

Section E.2

1. $\dfrac{1}{\tan^2 x}=\cot^2 x$
2. $\dfrac{}{1\quad \sin x}$

Section E.3

1. $\{\frac{\pi}{2}, \frac{3\pi}{2}\}$

2. $\{\frac{\pi}{3}, \frac{5\pi}{3}, \pi\}$

3. $3\ \frac{1}{\cos x}\ \ \frac{\sin x}{\cos x}\quad 2\quad 3\sin x\quad 2\cos^2 x\quad 3\sin x\quad 2(1\pi\pi\sin^2 x)$